

Critical coupling and coherent perfect absorption for ranges of energies due to a complex gain and loss symmetric system

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Abstract

We consider a non-Hermitian medium with a gain and loss symmetric, exponentially damped potential distribution to demonstrate different scattering features analytically. The condition for critical coupling (CC) for unidirectional wave and coherent perfect absorption (CPA) for bidirectional waves are obtained analytically for this system. The energy points at which total absorption occurs are shown to be the spectral singular points for the time reversed system. The possible energies at which CC occurs for left and right incidence are different. We further obtain periodic intervals with increasing periodicity of energy for CC and CPA to occur in this system.

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1 Introduction

The recent ideas of PT -symmetric non-Hermitian quantum mechanics [1, 2, 3] have been fruitfully extended to optics due to formal equivalence between Schroedinger equation and certain wave equations in optics [4, 5, 6]. The parity operator P stands for spatial reflections ($x \rightarrow -x, p \rightarrow -p$), while the anti-linear time reversal operator T leads to ($i \rightarrow -i, p \rightarrow -p, x \rightarrow x$). The equivalence between quantum mechanics and optics becomes possible due to the judicious inclusion of complex refractive index distribution $V(x) = \eta_R(x) + i\eta_I(x)$, in the electromagnetic wave equation [5, 7]. To realize this consider a one dimensional optical system with effective refractive index $\eta_R(x) + i\eta_I(x)$ in the background of constant refractive index η_0 , η_I stands for gain and loss component. The electric field $E = E_0(x, z)e^{i(\omega t - kz)}$ of a light wave propagating in this medium (with $\eta_0 \gg \eta_I, \eta_R$) satisfies the Schroedinger like equation,

$$\begin{aligned} i\frac{\partial}{\partial z}E_0(x, z) &= \left[\frac{1}{2k} \frac{\partial^2}{\partial x^2} + k_0[\eta_R(x) + i\eta_I(x)] \right] E_0(x, z) \\ &= HE_0(x, z) \end{aligned} \quad (1)$$

$k = k_0\eta_0$, k_0 being the wave number in vacuum. This Hamiltonian H has gain and loss symmetry for $\eta_R(x) = \eta_R(-x)$ and $\eta_I(x) = -\eta_I(-x)$. Thus complex optical gain and loss potential can be realized by judiciously incorporating gain and loss profile on an even index distribution. This important realization opens a wide window to study optical systems with gain and loss medium and leads to the experimental observation of PT -invariance and its breaking [8]-[17] in various optical systems [4, 5, 6, 7, 18, 19].

Various features of quantum scattering due to non-Hermitian potentials like exceptional points [20]-[22] and spectral singularity (SS) [23]-[26], reflectionlessness and invisibility [25, 27, 28, 29], reciprocity [25, 26, 30] etc have generated huge interest due to their applicability and usefulness in the study of optics. Recently the observation of perfect absorption [31]-[44] of incident electromagnetic wave by an optical media with complex refractive index distribution is counted as a big achievement in optics. The coherent perfect absorber (CPA) which is actually the time-reversed counterpart of a laser has become the center to all such studies in optics due to the discovery of anti-laser [31, 32, 33] in which incoming beams of light interfere with one another in such a way as to perfectly cancel each other out. This phenomena of perfect absorption in optics can also be observed in quantum scattering when particles (with a mass m and total energy E) interact with the surrounding medium through a complex potential distribution $V(x)$.

Scattering due to complex potential can be described in a simple mathematical language as follows. If A and B are the incident wave amplitudes from left and right directions and C and D are the outgoing wave amplitudes to left and right respectively, then the scattering amplitudes are related through scattering matrix as,

$$\begin{pmatrix} C \\ D \end{pmatrix} = S \begin{pmatrix} A \\ B \end{pmatrix}, \quad \text{where } S = \begin{pmatrix} t_l & r_r \\ r_l & t_r \end{pmatrix}. \quad (2)$$

For the perfect absorption the outgoing amplitudes C and D vanish leading to,

$$t_l A + r_r B = 0 ; \quad r_l A + t_r B = 0 ; \quad (3)$$

The condition for perfect absorption for unidirectional incident waves can be written as,

$$\begin{aligned} t_l = 0 ; r_l &= 0, \text{ for left incidence } (B = 0) \\ t_r = 0 ; r_r &= 0, \text{ for right incidence } (A = 0) \end{aligned} \quad (4)$$

These situations are known in literature as critical coupling (CC) [40]-[44]. On the other hand for bidirectional incident wave the condition for perfect absorption is,

$$|det[S]| = |t_l t_r - r_l r_r| = 0 \quad (5)$$

This condition refers as coherent perfect absorption which has recently created lots of excitement [31]-[39] due to the discovery of anti-laser. This could pave the way for a number of novel technologies with various applications from optical computers to radiology [31, 33]. Thus it is extremely important to investigate different aspects of CPA using different non-Hermitian systems.

The purpose of this work is to investigate various perspectives of scattering of particles due to a complex potential. In particular we would like to study different aspects of null scattering (CC and CPA) and super scattering in the context of a particular non-Hermitian gain and loss symmetric system which shows rich scattering properties. We consider non-Hermitian PT-symmetric version of an exponentially damped optical potential to derive the condition for CC and CPA analytically. This specific potential is rather known as Wood-Saxon (WS) [45, 46] potential, plays an important role in describing microscopic particle interactions. CPA never happens for Hermitian or PT-symmetric non-Hermitian systems as $|det[S]|$ is always unity in these cases. Here the complex PT-symmetric Wood-Saxon potential with an additive imaginary shift to the real axis shows scattering spectrum with total absorption for discrete as well as continuous ranges of energies. We further explicitly show the energy points at which CC or CPA occur correspond to SS points of the time reversed system. So far discrete energy points for CC and CPA are obtained for different optical media. We are able to obtain narrow but finite ranges of energies for CC and CPA to occur in this system. The nature of CC depends on the direction of incident wave. No energy range exists for right incidence. This happens due to the asymmetric left-right asymptotic behavior of this particular potential. More interestingly these ranges occur periodically with increasing periodicity for this particular system. This demonstration can give ideas to build perfect absorbers of matter waves which would be flexible to work for different ranges of energies of the incident particles. As the potential distribution is analogous to the medium's refractive index profile this work can be extended to quantum optics for the absorption of electromagnetic waves.

Now we present the plan of this paper. In section II we discuss the scattering of non-Hermitian PT-symmetric WS potential and its time reversal partner potential. The CC for this system at discrete positive energies is reported in Sec. III. In Sec. IV we calculate the periodic ranges for CC in this potential. Sec. V is devoted to obtain the condition for CPA and its ranges while Sec. VI is left for conclusions and discussions.

2 Scattering from gain and loss symmetric WS potential

In this section we explore the nature of scattering when the single-particle space is expanded in a Wood-Saxon basis [47]. If particle scatters through a medium consists of semi-infinite nuclear matter it is more reasonable to take the nucleon distribution of a Wood-Saxon type rather than an uniform distribution [48, 49]. The Hermitian WS potential in a simplified form [50] can be written as ,

$$V(x) = -\frac{V_0}{1 + e^{\frac{x-r_0}{a}}} \equiv -\frac{V_0}{1 + qe^{\delta x}} , \quad (6)$$

where V_0 is the WS potential depth, r_0 is the width of the potential i.e the nuclear radius and a is the diffuseness of semi-infinite nuclear matter [48]. The WS-potential is rewritten in a more simplified form in Eq. (6) with $\delta = 1/a$ and $q = e^{-r_0\delta}$, for its quantum mechanical study. In this simplified form V_0 and δ determines the height and shape of the potential respectively. For positive δ and V_0 this potential asymptotically becomes $-V_0$ for $x \rightarrow -\infty$ and vanishes for $x \rightarrow +\infty$. The solutions of Schroedinger equation for this potential correspond to both bound states and scattering states and are useful in studying different problems. Two independent scattering state solutions of the particle wave equation (with $q = 1$ and $\hbar = 1$) for WS potential are written as [50],

$$\begin{aligned} \psi_1(x) &= M(1 + e^{\delta x})^{-\alpha_2 - \alpha_3} e^{\alpha_3 \delta x} \\ &\times {}_2F_1(1 + \alpha_2 + \alpha_3, \alpha_2 + \alpha_3; 1 + 2\alpha_2; \frac{1}{1 + e^{\delta x}}) ; \end{aligned} \quad (7)$$

$$\text{and } \psi_2(x) = M(\psi_1(x) |_{\alpha_2 \rightarrow -\alpha_2}) , \quad M = (-1)^{2\alpha_3} \frac{\Gamma(\alpha_2 - \alpha_3)\Gamma(1 + \alpha_2 - \alpha_3)}{\Gamma(1 + 2\alpha_2)\Gamma(-2\alpha_3)} , \quad (8)$$

$$\text{with } \alpha_2 = \frac{2i}{\delta} \sqrt{mE} \equiv \frac{2i}{\delta} k_1; \quad \alpha_3 = \frac{2i}{\delta} \sqrt{m(E + V_0)} \equiv \frac{2i}{\delta} k_2 . \quad (9)$$

One can calculate the different scattering amplitudes by considering the superposition of these two independent scattering state solutions and looking at the asymptotic behaviors of these. We complexify the WS potential by taking the shape parameter imaginary (i.e. $\delta \rightarrow i\rho$) such that it becomes PT-symmetric. Moreover for latter convenience we would like to consider this complex potential with an imaginary shift to the real axis as [51, 52],

$$\tilde{V}(\bar{x}) = -\frac{V_0}{1 + \bar{q}e^{i\rho\bar{x}}} , \quad \text{where } \bar{x} = x - i\zeta , \quad \zeta \text{ is real,} \quad (10)$$

where $\bar{q} = e^{-ir_0\rho} = 1$ and the complex part of the potential $\tilde{V}(\bar{x})$ working as the gain and loss component for this complex system.

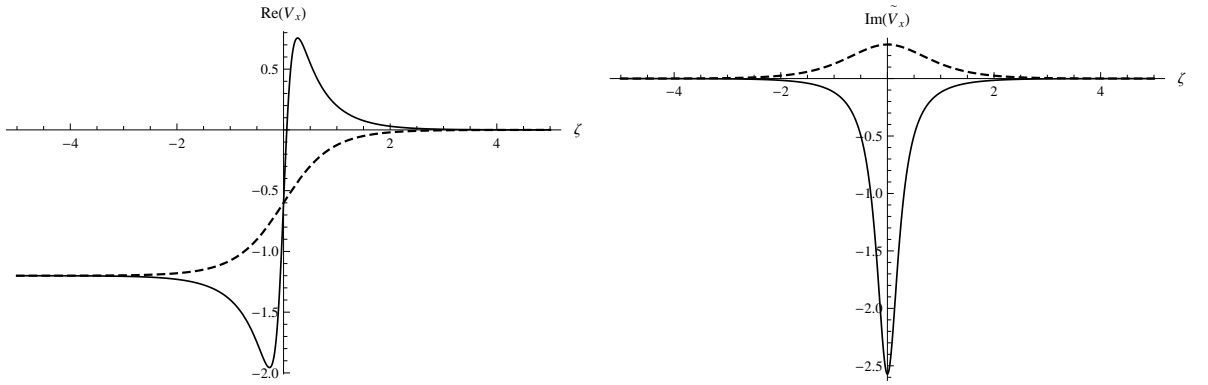


Fig. 1: Distributions of the real and imaginary components of PT -symmetric complex potential $\tilde{V}(\bar{x})$ are shown against ζ (for $V_0 = 1.2$ and $\rho = 1.8$) with the values $x = 2$ (continuous lines) and $x = 4$ (dashed lines).

The Schroedinger equation with respect to \bar{x} will have the same form as with respect to real x . In this case x bears the periodic nature of $\tilde{V}(\bar{x})$ where ζ decides the asymptotic behavior of the potential. The complexified WS potential in Eq. (10) has the same asymptotic behaviors as $\{x, \zeta\} \rightarrow +\infty$ and $\{x, \zeta\} \rightarrow -\infty$ shown in Fig. 1. This observation leads us to obtain the scattering state solutions of PT -symmetric non-Hermitian WS potential in the same form as in the Eq. (7) and (8) subjected to the following modifications,

$$\begin{aligned} x \rightarrow \bar{x} ; \quad \alpha_2 \rightarrow a_2 &= \frac{2}{\rho} \sqrt{mE} \equiv \frac{2}{\rho} k_1 ; \\ \alpha_3 \rightarrow a_3 &= \frac{2}{\rho} \sqrt{m(E + V_0)} \equiv \frac{2}{\rho} k_2 . \end{aligned} \quad (11)$$

The above solutions for the non-Hermitian case have the following asymptotic behaviors,

$$\begin{aligned} \psi_1(\bar{x} \rightarrow +\infty) &= M e^{-ik_1 \bar{x}} e^{-k_1 \zeta} = M e^{-ik_1 \bar{x}} ; \\ \psi_1(\bar{x} \rightarrow -\infty) &= M [G1 e^{ik_2 \bar{x}} e^{k_2 \zeta} + G2 e^{-ik_2 \bar{x}} e^{-k_2 \zeta}] = M [G1 e^{ik_2 \bar{x}} + G2 e^{-ik_2 \bar{x}}] ; \\ \psi_2(\bar{x} \rightarrow +\infty) &= M e^{ik_1 \bar{x}} e^{k_1 \zeta} = M e^{ik_1 \bar{x}} ; \\ \psi_2(\bar{x} \rightarrow -\infty) &= M [G3 e^{ik_2 \bar{x}} e^{k_2 \zeta} + G4 e^{-ik_2 \bar{x}} e^{-k_2 \zeta}] = M [G3 e^{ik_2 \bar{x}} + G4 e^{-ik_2 \bar{x}}] , \end{aligned} \quad (12)$$

where $G1$, $G2$, $G3$, and $G4$ are given in terms of Gamma functions as,

$$\begin{aligned} G1 &= \frac{\Gamma(1 + 2a_2) \Gamma(-2a_3)}{\Gamma(a_2 - a_3) \Gamma(1 + a_2 - a_3)} ; \\ G2 &= \frac{\Gamma(1 + 2a_2) \Gamma(2a_3)}{\Gamma(a_2 + a_3) \Gamma(1 + a_2 + a_3)} ; \end{aligned}$$

$$\begin{aligned}
G3 &= \frac{\Gamma(1-2a_2)\Gamma(-2a_3)}{\Gamma(-a_2-a_3)\Gamma(1-a_2-a_3)}; \\
G4 &= \frac{\Gamma(1-2a_2)\Gamma(2a_3)}{\Gamma(-a_2+a_3)\Gamma(1-a_2+a_3)}.
\end{aligned} \tag{13}$$

The scattering wavefunctions from the solutions in Eq. (12) asymptotically diverges as the potential becomes 0 and $-V_0$ for $\zeta \rightarrow \pm\infty$ in the Fig. 1. Different scattering amplitudes for this gain and loss symmetric complex WS potential thus can be readily read out as,

$$r_l = \frac{G4}{G3} = \frac{\Gamma(2a_3)\Gamma(-a_2-a_3)\Gamma(1-a_2-a_3)}{\Gamma(-2a_3)\Gamma(-a_2+a_3)\Gamma(1-a_2+a_3)}; \tag{14}$$

$$t_l = \sqrt{\frac{k_1}{k_2}} \frac{1}{G3} = \sqrt{\frac{k_1}{k_2}} \frac{\Gamma(-a_2-a_3)\Gamma(1-a_2-a_3)}{\Gamma(1-2a_2)\Gamma(-2a_3)} = t_r; \tag{15}$$

$$r_r = -\frac{G1}{G3} = -\frac{\Gamma(1+2a_2)\Gamma(-a_2-a_3)\Gamma(1-a_2-a_3)}{\Gamma(1-2a_2)\Gamma(a_2-a_3)\Gamma(1+a_2-a_3)}. \tag{16}$$

It is easy to see that $|r_l|^2 \equiv R_l \neq R_r \equiv |r_r|^2$ as a_2, a_3 are real, as expected for PT-symmetric non-Hermitian systems. However we have $|t_l|^2 \equiv T_l = T_r \equiv |t_r|^2$ in this case unitarity is violated, $R+T \neq 1$. On the other hand reciprocity and unitarity are restored for Hermitian case as $G1^* = G4$ and $G2^* = G3$, since α_2 and α_3 are purely imaginary.

Under time reversal transformation the WS potential becomes,

$$\tilde{V}^*(\bar{x}) = -\frac{V_0}{1 + e^{-i\rho\bar{x}^*}}. \tag{17}$$

The scattering amplitudes for $\tilde{V}^*(\bar{x})$ (denoted with prime) are obtained from that of $\tilde{V}(\bar{x})$ by changing the parameter $\rho \rightarrow -\rho$ i.e. $a_2 \rightarrow -a_2$ and $a_3 \rightarrow -a_3$ as,

$$r'_l = r_l \Big|_{\substack{a_2 \rightarrow -a_2 \\ a_3 \rightarrow -a_3}} = \frac{\Gamma(-2a_3)\Gamma(a_2+a_3)\Gamma(1+a_2+a_3)}{\Gamma(2a_3)\Gamma(a_2-a_3)\Gamma(1+a_2-a_3)}; \tag{18}$$

$$t'_l = t_l \Big|_{\substack{a_2 \rightarrow -a_2 \\ a_3 \rightarrow -a_3}} = \sqrt{\frac{k_1}{k_2}} \frac{\Gamma(a_2+a_3)\Gamma(1+a_2+a_3)}{\Gamma(1+2a_2)\Gamma(2a_3)} = t'_r; \tag{19}$$

$$r'_r = r_r \Big|_{\substack{a_2 \rightarrow -a_2 \\ a_3 \rightarrow -a_3}} = -\frac{\Gamma(1-2a_2)\Gamma(a_2+a_3)\Gamma(1+a_2+a_3)}{\Gamma(1+2a_2)\Gamma(a_2-a_3)\Gamma(1-a_2-a_3)}. \tag{20}$$

Different properties of null scattering and super scattering will be discussed using these scattering amplitudes in the following sections.

3 Total absorption of unidirectional wave: Critical Coupling

Particle waves of certain specific frequencies when incident from one direction on a potential are completely absorbed by the potential due to critical coupling (CC). In this section we discuss the CC due to non-Hermitian gain and loss symmetric WS potential and calculate the frequencies which are absorbed by the system. The transmission and reflection coefficients become identically zero at these energies. We observe that the frequencies of the waves for CC for left incidence are different from that of for right incidence.

3.1 CC for left incident wave

The transmission and reflection coefficients of $\tilde{V}(\bar{x})$ are written from Eqs. (14) and (15) as,

$$R_l = |r_l|^2 = \left| \frac{G4}{G3} \right|^2 ; T_l = |t_l|^2 = \left| \frac{1}{G3} \right|^2 . \quad (21)$$

R_l and T_l become simultaneously zero if $2a_3 = n$, a positive integer. This happens for the positive discrete energy,

$$E_n^l = \frac{\rho^2}{16m} n^2 - V_0, \text{ with } n > \frac{4}{\rho} \sqrt{mV_0} . \quad (22)$$

This physically means when matter wave with energy E_n^l is incident on a medium with potential distribution $\tilde{V}(\bar{x})$ then the wave will be completely absorbed as $R_l = 0$ and $T_l = 0$. The successive energy separations for this null scattering,

$$\Delta E_n^l \equiv E_{n+1}^l - E_n^l = \frac{\rho^2}{16m} (2n + 1) \quad (23)$$

are independent of depth of the potential. Now we consider the time reversed case of $\tilde{V}(\bar{x})$ with reflection and transmission amplitudes $r'_{l,r}$ and $t'_{l,r}$ as given in Eqs. (18), (20) and (19). In this case R'_l diverges when $2a_3 = n$, leading to the SS at the same energy point E_n^l . Thus we analytically see CC of a gain and loss symmetric non-Hermitian WS potential are the SS points of the time reversed of the same potential. This is demonstrated graphically in Fig. 2.

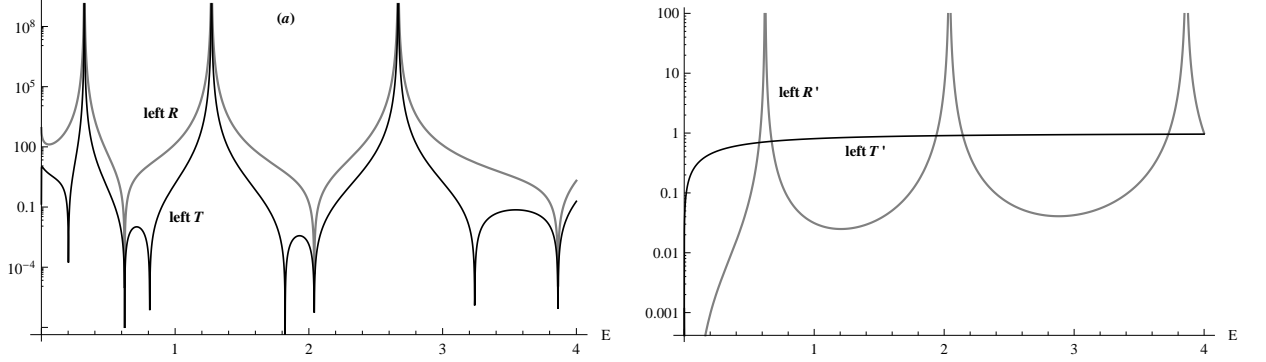


Fig. 2: Critical couplings for left incident waves against incident energies are shown for the potential $\tilde{V}(\bar{x})$ in Fig. 2(a) and its time reversal partner $\tilde{V}^*(\bar{x})$ in Fig. 2(b). In Fig. 2(a) both R_l and T_l vanish at the energies E_n^l with $n_{min} = 3$ (for $V_0 = 1.2, \rho = 1.8$ and $m = 1$). On the other hand Fig. 2(b) shows divergence of total scattering coefficient (as $R_l' \rightarrow \infty$) at the same incident energies for the time reversed potential.

3.2 CC for right incident wave

In this subsection we would like to emphasize that condition for CC depends on the direction of incidence. In particular we show that condition of CC for left incidence is different from that of for right incidence for gain and loss symmetric non-Hermitian WS potential. From Eqs. (15) and (16) the scattering coefficients $R_r = |r_r|^2 = |\frac{G1}{G3}|^2$ and $T_r = |t_r|^2 = |\frac{1}{G3}|^2$ vanish simultaneously when $2a_2$ is a positive integer (n'). This leads to CC at the energy, $E_{n'}^r = \frac{\rho^2}{16m}n'^2$, which is different from E_n^l at which CC occurs for left incidence. This happens due to the asymmetry in the left and right asymptotic behavior of the potential. However the energy separation between two consecutive CC is independent of direction of incident wave for same values of n and n' as,

$$\Delta E_{n'}^r \equiv E_{n'+1}^r - E_{n'}^r = \frac{\rho^2}{16m}(2n' + 1) = \Delta E_{n=n'}^l. \quad (24)$$

Following the discussion of the previous subsection it can be shown easily that null scattering due to $\tilde{V}(\bar{x})$ is same as the super scattering due to $\tilde{V}^*(\bar{x})$ at the same energy even for right incidence. This situation is nicely demonstrated in Fig. 3.

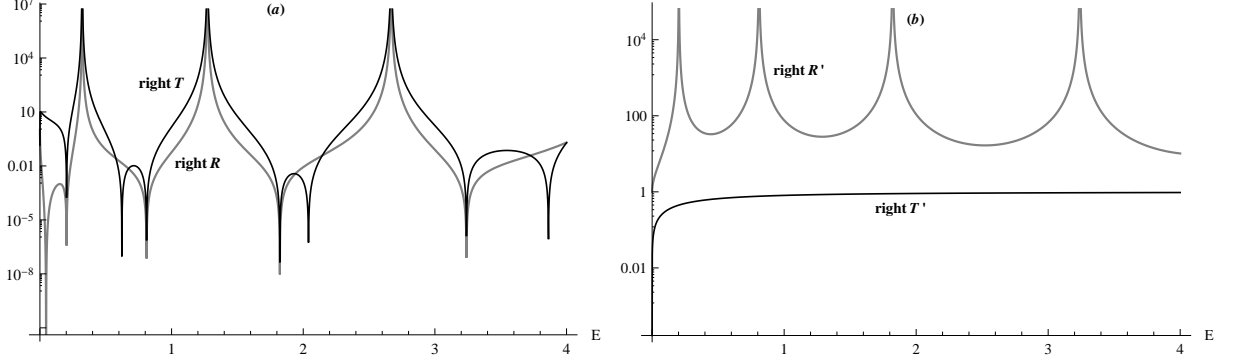


Fig. 3: Critical couplings for right incidence shown for gain and loss symmetric WS potential. In Fig. 3(a) both R_r and T_r for $\tilde{V}(\bar{x})$ vanish at the energies E_n^r with $n = 1, 2, 3$. (for $V_0 = 1.2, \rho = 1.8$ and $m = 1$). On the other hand Fig. 3(b) shows the divergences of R'_r at the same incident energies for $\tilde{V}^*(\bar{x})$.

Thus the gain and loss symmetric WS potential can work as a critical coupler for both left and right incidences but for different frequencies. The waves with these frequencies when incident from left or right on the time reversed gain and loss symmetric WS potential result in super reflectivity ($R'_l \rightarrow \infty$ or $R'_r \rightarrow \infty$, with finite T').

4 Critical coupling for ranges of incident energies

In this section we find critical coupling for different energy ranges. To demonstrate this we would like to consider time reversed potential $\tilde{V}^*(\bar{x})$ for which the transmission and reflection amplitudes are written in Eqs. (18)-(20). For this time reversed case left handed transmission amplitude neither vanishes nor diverges as a_2, a_3 are real positive numbers. However we can adjust the values of the parameter ρ and V_0 in such a manner that $T' \equiv |t'_{l,r}|^2$ becomes negligibly small over an interval of energy. This is achieved by considering ρ very small such that a_2 and a_3 are very high and by adjusting V_0 in such a manner that $a_3 \gg a_2$. In that case the dominating term $\Gamma(2a_3)$ in the denominator of the transmission amplitude will be very high leading to very small or negligible transmission. At the same time left handed reflection amplitude also becomes negligible for a certain interval of energy due to the presence of $\Gamma(2a_3)$ term in denominator. Simultaneously $R'_l \equiv |r'_l|^2$ also vanishes for discrete positive energies at which $a_2 - a_3 = -n$ condition is satisfied. But this interval is interrupted by certain singularities when $2a_3 = N$ occur. Both n and N are positive integers. The positive discrete energies at which R'_l diverges are written as,

$$E_{ss}^N = \frac{N^2 \rho^2}{16m} - V_0, \quad \text{for } N > \frac{4}{\rho} \sqrt{mV_0}. \quad (25)$$

Energy gap between two such successive SS is,

$$\Delta E_{ss} = E_{ss}^{N+1} - E_{ss}^N = (2N + 1) \frac{\rho^2}{16m} . \quad (26)$$

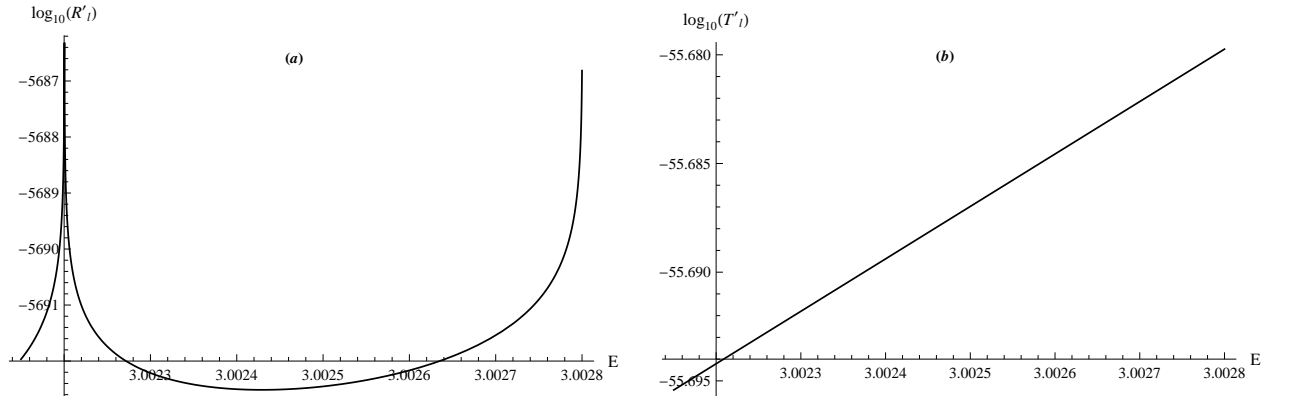
Therefore between two consecutive spectral singularities we have a certain energy range where $R'_l \approx 0, T'_l \approx 0$. This range is linearly increasing with the integer values of N , and also can be controlled by the parameters ρ and V_0 . In such intervals of energy R'_l becomes exactly zero at certain discrete positive energies (for $a_2 - a_3 = -n$) which are calculated as,

$$\begin{aligned} E - \sqrt{E^2 + EV_0} &= \frac{1}{2} \left(\frac{n\rho}{2\sqrt{m}} \right)^2 - \frac{V_0}{2} \equiv p_n , \\ \text{i.e. } E_n &= \frac{p_n^2}{V_0 + 2p_n} . \end{aligned} \quad (27)$$

These discrete energies E_n is real and positive for $n > \sqrt{\frac{4mV_0}{\rho^2}}$. The separation between two such consecutive zeros of R'_l is computed as,

$$E_{n+1} - E_n = (2n + 1) \left(\frac{\rho^2}{18m} - \frac{mV_0^2}{n^2(n+1)^2\rho^2} \right) . \quad (28)$$

Thus in case of $\tilde{V}^*(\bar{x})$ we obtain different range of CC separated by SS for left incidence. On the other hand for $\tilde{V}(\bar{x})$ we obtain ranges of SS separated by CC at the same energy values. All these features of WS potential are well illustrated in Fig. 4. However we would like to point out no such ranges of energy exist for $\tilde{V}^*(\bar{x})$ in the case of right incidence.



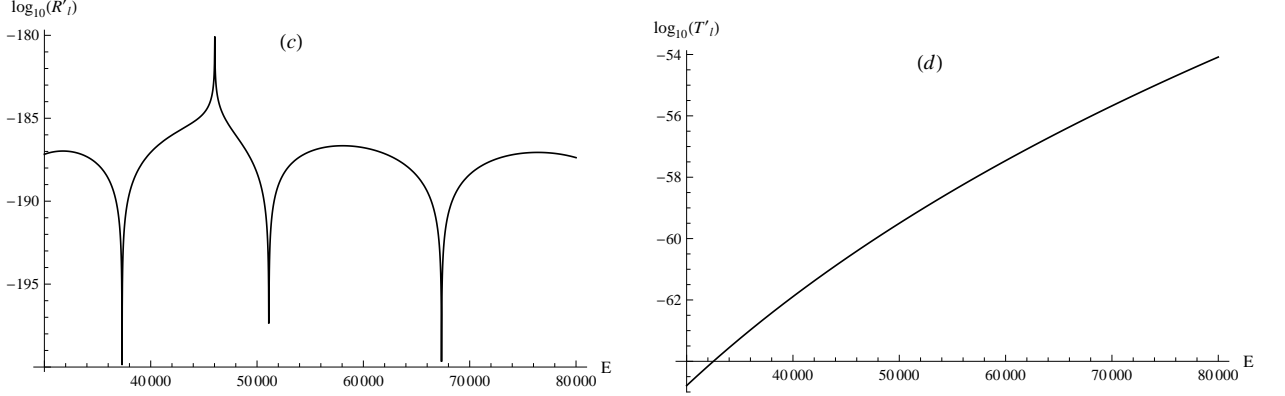


Fig. 4: $\log_{10}(R'_l)$ and $\log_{10}(T'_l)$ are plotted to demonstrate different energy ranges of CC due to the potential $\tilde{V}^*(\bar{x})$. In Figs. 4(a) and (b) one particular range of energy is shown between two successive SS ($\rho = .0006, V_0 = 1, m = 1$) where $R'_l \approx 0$ and $T'_l \approx 0$. Different energy ranges for $R'_l \approx 0$ are shown in Fig. 4(c) (for $\rho = 60; V_0 = 5.5 \times 10^6, m = 1$), whereas Fig. 4(d) shows that T'_l is vanishingly small in those intervals. Any desired ranges of incident energy for CC can be achieved with the potential $\tilde{V}^*(\bar{x})$ by adjusting the parameters in the potential.

5 Perfect absorption of bidirectional waves

In this section we investigate the absorption through this particular potential distribution when waves are coming from both directions. As mentioned in the introduction perfect absorption for bidirectional waves will occur if $|t_l t_r - r_l r_r| = 0$. From Eqs. (14)-(16) we obtain,

$$\frac{1}{(G3)^2} \left[\frac{k_1}{k_2} + G1G4 \right] = 0. \quad (29)$$

Using the properties of Γ function we find the relation $G4G1 + \frac{k_1}{k_2} = G2G3$. Thus perfect absorption occurs for the bidirectional waves if $\frac{G2}{G3} = 0$ ($G3 \neq 0$). Since a_2, a_3 are real positive numbers, CPA will occur for the potential $\tilde{V}(\bar{x})$ when $G3$ is infinity ($G2$ can not be zero for $\tilde{V}(\bar{x})$). From Eq. (13) we see that this situation occurs in two possible ways either $2a_2 = n_1 + 1$ or $2a_3 = n_2$. Thus we obtain CPA for $\tilde{V}(\bar{x})$ at two different energies

$$E_{n_1} = \frac{(n_1 + 1)^2 \rho^2}{16m} \quad \text{and} \quad E_{n_2} = \frac{n_2^2 \rho^2}{16m} - V_0, \quad (30)$$

where n_1, n_2 are positive integers. At energies E_{n_1} , $T = 0, R_l = 0$ and at energies E_{n_2} , $T = 0, R_r = 0$ [as shown in Fig. 5]. Alternatively for the time reversed case a_2 and a_3 changes sign and CPA can only occur for $G2 = 0$ ($G3$ is always finite in the case of $\tilde{V}^*(\bar{x})$).

The discrete positive energies for CPA are calculated as,

$$E + \sqrt{E^2 + EV_0} = \frac{M^2 \rho^2}{8} - \frac{V_0}{2} \equiv q_M ,$$

$$\text{i.e. } E = E_M^* = \frac{q_M^2}{V_0 + 2q_M} \text{ (where } M \text{ is positive integer).} \quad (31)$$

To ensure real positive energy M must be greater than $\sqrt{\frac{4mV_0}{\rho^2}}$. This depicts that at least one such discrete positive energy for CPA will exist for the potential $\tilde{V}^*(\bar{x})$ if $\rho^2 > \frac{|V_0|}{4}$. At these energies left incident and right incident waves interfere destructively and absorbed by the potential completely. Waves with energies E_M^* when incident on the potential $\tilde{V}(\bar{x})$ produce lasing behavior where all the scattering amplitudes diverge (as $a_2 + a_3$ is equal to a positive integer). Similarly for $\tilde{V}^*(\bar{x})$, $R'_l \rightarrow \infty$ and $R'_r \rightarrow \infty$ at energies E_{n_1} and E_{n_2} respectively where CPA occur for $\tilde{V}(\bar{x})$. Thus spectral singularity of $\tilde{V}^*(\bar{x})$ and CPA of $\tilde{V}(\bar{x})$ occur at the same energy points.

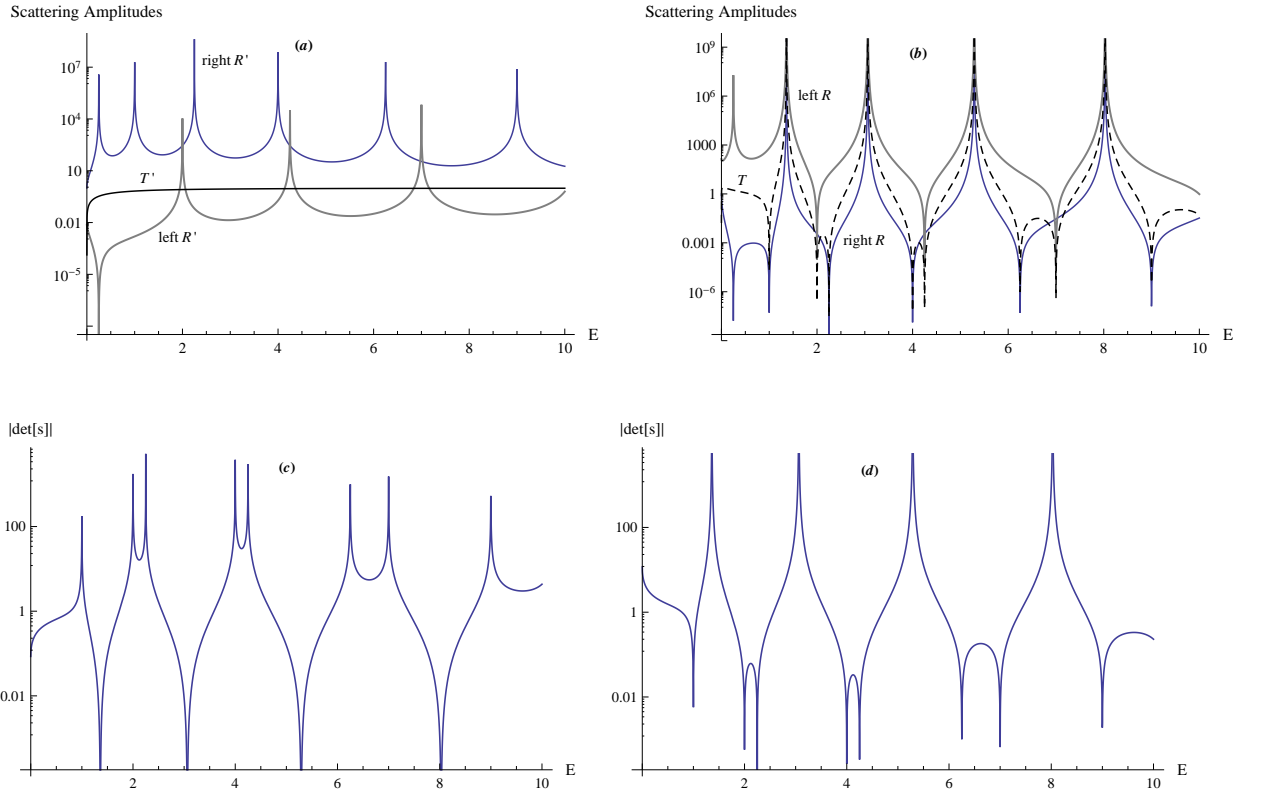


Fig. 5: Figs (a) and (c) are for the potentials $\tilde{V}^*(\bar{x})$ whereas (b) and (d) are for $\tilde{V}(\bar{x})$. Behavior of R and T are shown in (a) and (b) for $V_0 = 2, \rho = 2, m = 1$. On the other hand CPA and its time reversed situation are shown in (c) and (d).

To obtain the ranges of energy for CPA for the potential $\tilde{V}^*(\bar{x})$ we choose a small ρ such that a_2, a_3 are large even for lower energies. Then we adjust V_0 in such a manner that a_3 is large enough compare to a_2 . In such situation we obtain a range of energy for which $t'_{r,l} \approx 0$ and $r'_l \approx 0$ and hence $|\det[S]| \approx 0$, leading to CPA. This range is interrupted by the singular points of r'_l and r'_r which occurs due to the presence of $\Gamma(-2a_3)$ and $\Gamma(1-2a_2)$ in the numerator of r'_l and r'_r respectively. Thus range of CPA is separated by these SS as shown in Fig. 6. The n^{th} spectral singular point for r'_l is at the positive discrete energies $E_n = \frac{n^2 \rho^2}{16m} - V_0$ for which $2a_3 = n$, with $n_{\min} = \text{Int}(\sqrt{\frac{16mV_0}{\rho^2}}) + 1$. The energy interval for any two consecutive singularities of r'_l is,

$$\Delta E_{ss}^{rl} = E_{n+1} - E_n = \frac{\rho^2}{16m}(1 + 2n).$$

On the other hand r'_r has singularities for $2a_2 = n'$ (i.e. at $E'_n = \frac{n'^2 \rho^2}{16m}$, where n' is another positive integer) which occur at the energy intervals of,

$$\Delta E_{ss}^{rr} = E'_{n'+1} - E'_n = \frac{\rho^2}{16m}(1 + 2n').$$

Both the energy intervals occurs periodically and $\Delta E_{ss}^{rl} > \Delta E_{ss}^{rr}$ as $n > n'$. The energy span of these ranges increase with increasing n' for a fixed ρ . Fig. 6 shows the ranges of energies of perfect absorption for the time reversed potential with different parametric regimes.

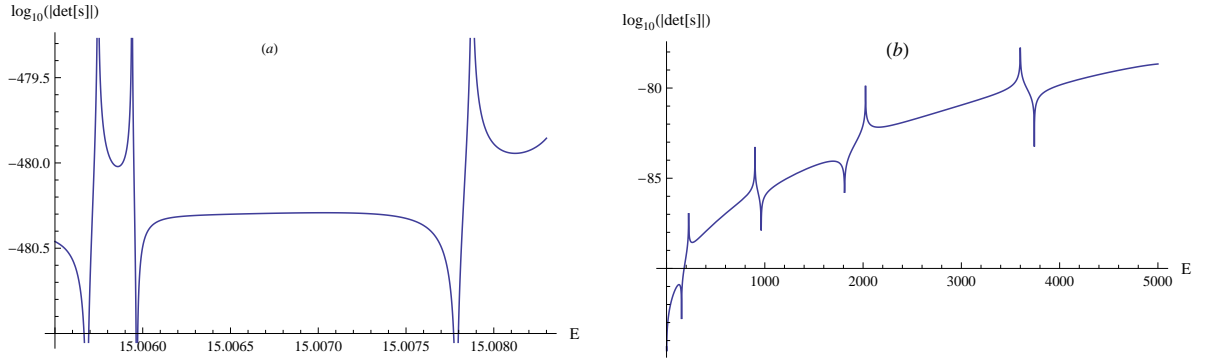


Fig. 6: Energy ranges for CPA for the potential $\tilde{V}^*(\bar{x})$ are demonstrated. Fig. 6(a) shows a range of energy with $\rho = .001, V_0 = 15, m = 1$. In Fig. 6(b) we have shown periodic ranges of energies separated by spectral singular points for a different parametric regime ($\rho = 60; V_0 = 5.5 \times 10^6, m = 1$).

6 Conclusions and discussion

It worths studying CC and CPA and its properties for interaction of particle waves with various non-Hermitian models to search for new features and possibilities for perfect ab-

Table 1: Energy and energy ranges for CC and CPA for different values of parameters (with incident particle mass $m = 1$, in atomic unit) of non-Hermitian PT-symmetric WS potential.

V_0	$a = 1/\rho$ (in nm)	$r_0 = \pi.a$ (in nm)	Incident particle energies & energy ranges for CC and CPA	Corresponding Figs.
32.65 eV	.029	.093	$E_3^l = 16.94$ eV, $E_4^l = 55.50$ eV, $E_5^l = 105.07$ eV.	Fig. 2
32.65 eV	.029	.093	$E_1^r = 5.51$ eV, $E_2^r = 22.04$ eV, $E_3^r = 49.58$ eV.	Fig. 3
27.2 eV	88.3	277.5	Range of CC $\cong .0006$ eV; (E=81.6791 eV to 81.6954 eV)	Fig. 4 (a,b)
150 MeV	.0009	.003	Range of CC $\cong .46$ MeV. (E=1.37 MeV to 1.83 MeV)	Fig. 4 (c,d)
54.41 eV	.02	.08	$E_{n1} = 27.2$ eV($n1 = 1$), $E_{n2} = 54.4$ eV ($n2 = 4$), $E_{n1} = 61.2$ eV($n1 = 2$).	Fig. 5(b,d)
54.41 eV	.02	.08	$E_M^* = 37.03$ eV($M = 3$), $E_M^* = 83.32$ eV($M = 4$), $E_M^* = 143.92$ eV($M = 5$).	Fig. 5(a,c)
408.01 eV	53	166.5	Range of CPA $\cong .053$ eV; (E=408.096 eV to 408.258 eV)	Fig. 6(a)
150 MeV	.0009	.003	Range of CPA $\cong .042$ MeV. (E=.055 MeV to .097 MeV)	Fig. 6(b)

sorption. In this work we have shown that for a particular gain and loss symmetric non-Hermitian optical potential (WS potential) it is possible to achieve CC and CPA for a range of frequencies due to quantum scattering. We have obtained that the conditions of CC depend on the direction of incident waves and no range exists for right incident case for this non-Hermitian potential. More interestingly by adjusting the parameters in the potential we can have these total absorptions in any desired ranges of frequencies. For a PT-symmetric non-Hermitian optical potential we have derived the analytical conditions of null scattering (CC and CPA) to occur. The energy points at which the null scattering occurs are shown to be the SS points for the time reversed system. An estimation of the values of parameters are provided in in Table. 1. The CC and CPA energies and energy ranges increases with increasing shape parameter, i.e. decreasing diffuseness of the potential. As this specific potential plays an important role in describing interactions of nucleon with heavy nucleus [45, 46], our theoretical demonstration will open the possibility of future work on absorption of interacting microscopic particles of different masses and energies.

Acknowledgments: Two of the authors (A. Ghatak and B. P. Mandal) acknowledge the hospitality of TIFR, Mumbai, where a part of revision of this manuscript is done.

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